

Prove the identities.

1. $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$

2. $\tan x + \sec x = \frac{\cos x}{1 - \sin x}$

3. $\frac{\sec^2 x - 1}{\sin x} = \frac{\sin x}{1 - \sin^2 x}$

4. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

5. $\frac{\sin t}{1 - \cos t} + \frac{1 + \cos t}{\sin t} = \frac{2(1 + \cos t)}{\sin t}$

6. $\sin^3 x \cos^3 x = (\sin^3 x - \sin^5 x)(\cos x)$

7. $\frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x} = \frac{1 - 4\cos x}{1 - \cos x}$

8. $\frac{1}{\tan \beta} + \tan \beta = \sec \beta \csc \beta$

Find all solutions on the interval of $[0, 2\pi)$.

9. $2\cos x \sin x - \cos x = 0$

10. $\sqrt{2}\tan x \cos x - \tan x = 0$

11. $3\cot^2 x - 1 = 0$

12. $\sec x \csc x = 2\csc x$

Find all solutions.

13. $4\cos^2 x - 4\cos x + 1 = 0$

14. $3\sin t = 2\cos^2 t$

15. $2\sin^2 x + 3\sin x + 1 = 0$

16. $2\cos 2x = 1$

17. $\cos 2x = -1$

18. $\tan^2 3x = 3$

19. $\cos 2x(2\cos x + 1) = 0$

20. $\tan \frac{x}{3} = 1$

21. $\csc^2 2x = 3\csc 2x + 4$

Use a graphing utility to approximate the solutions (to three decimal places) on the interval $[0, 2\pi)$.

22. $2\sin x + \cos x = 0$

23. $\csc^2 x + 0.5\cot x = 5$

24. $4\cos^2 x - 2\sin x + 1 = 0$